Decomposition Techniques for Large Scale Optimisation Problems

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UNIKASSEL VERSITÄT

Outline

Outline

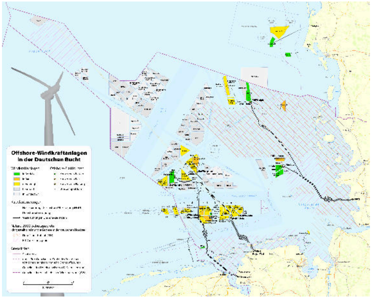
Wind Park Extension Planning

Optimisation Model

Lagrangian Solution Approach

Examples

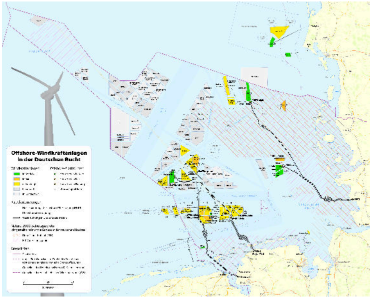
Construction Planning for Off-Shore Wind Parks



Static planning

- Which (new) off-shore wind parks and hub nodes should be built?
- Which connections should be built?
- What technical equipment should be installed? (transformers, converters, cable types, ...)

Construction Planning for Off-Shore Wind Parks



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- Meet supply goals, climate targets, budget constraints, . . .

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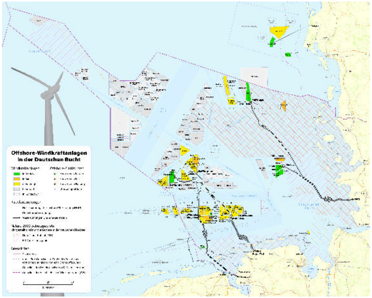

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Mathematical challenges

- Large networks,
- Complex technologies,
- Multi-period planning,

Assumptions

Simplifying assumptions for this talk:

- Only one construction step
- Constructed wind parks are known in advance
- Fixed set of possible technologies (cable types) for each connection
- Fixed set of possible technologies (transformer, converter) for each hub node
- Neglect technical details of energy transport, restrict to simple linear flows

$\Rightarrow$ Combined network design and capacity dimensioning problem.

Problem

Decisions to make

- Which connections and hub nodes should be built?
- Which technology to install?
- Capacity to be installed

Objective

- Minimise total cost

Constraints

- Capacity sufficient to transport energy to on-shore nodes
- Technologies on hub nodes and connections match

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Mathematical Model

Given

- Graph $G = (V, E)$ with

$$\begin{aligned} V = & V_W && \ldots \text{wind parks (producer)} \\ & \cup V_H && \ldots \text{hubs} \\ & \cup V_O && \ldots \text{on-shore (consumer)} \\ E & && \ldots \text{potential power lines} \end{aligned}$$

- Energy production / consumption for each node

Topology Design

- build which hubs $x_v \in \{0, 1\}$?
- build which connections $z_e \in \{0, 1\}$?

Dimensioning

- How many transformers $y_{t,v} \in \mathbb{N}_0$
- How many cables $u_{t,e} \in \mathbb{N}_0$

of technology $t \in T$?

Minimize Total Cost

$$\underbrace{\sum_{v \in V_H} c_v x_v}_{\text{hub opening}} + \underbrace{\sum_{e \in E} c_e z_e}_{\text{connection opening}} + \underbrace{\sum_{v \in V_H} \sum_{t \in T_v} c_{t,v} y_{t,v}}_{\text{transformer installation}} + \underbrace{\sum_{e \in E} \sum_{t \in T_e} c_{t,e} u_{t,e}}_{\text{cable installation}} \rightarrow \text{minimise}$$

Full Model

$$\min \quad \sum_{v \in V_H} c_v x_v + \ldots + \sum_{e \in E} \sum_{t \in T_e} c_{t,e} u_{t,e}$$

Objective

- Setup Costs
- Installation Costs

$$\text{subject to} \quad \boxed{AX \leq b,}$$

Constraints

- Transformers only on built hubs
- Cables only on built connections
- Sufficient capacity for transporting energy to on-shore nodes

$$X = (x, y, z, y)$$

Decision Variables

- Integral Variable Domain

Solution Approach

Standard Approach

- Model is a ***Mixed-Integer Linear Program***
- Solve model using state-of-the-art solvers (CPLEX, Gurobi, . . .)
  - Models often too large
  - Solving times too long

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Goals

- Exact solutions usually not required (data is inexact anyway)
  - Short computation times
  - Find sufficiently "good" solutions
  - Find ***bounds*** that ***certify the quality*** of the solution

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One very successful approach:
Lagrangian relaxation

Outline

General Approach

Idea

- Model of the form

$$
\begin{aligned}
\min \quad & f(x) \\
\text{subject to} \quad & x \in X, && \ldots\text{“easy” constraints} \\
& \left.\begin{aligned} Ax &= b, \\ Dx &\le d \end{aligned}\right\} && \ldots\text{“complicated” constraints}
\end{aligned}
$$

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- remove "complicated" constraints,

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- remove “complicated” constraints,
- penalise their violation,

$$L(\lambda, \mu) := \min \{ f(x) + \langle b - Ax, \lambda \rangle + \langle d - Dx, \mu \rangle : x \in X \}$$

Lagrangian function with ***multipliers*** $\lambda \in \mathbb{R}^m$, $\mu \le 0$

Lagrangian Decomposition

Important property:

- problem may decompose into ***independent***, ***smaller*** subproblems:

$$
\begin{array}{ll}
\min & f_1(x_1) + f_2(x_2) \\
\text{s.t.} & x_1 \in X_1, x_2 \in X_2, \\
 & A_1 x_1 + A_2 x_2 = b, \\
 & D_1 x_1 + D_2 x_2 \leq d,
\end{array}
$$

$$
\langle b, \lambda \rangle + \langle d, \mu \rangle
$$

constant

$$
\begin{array}{ll}
\min & f_1(x_1) - \langle A_1^T \lambda + D_1^T \mu, x_1 \rangle \\
\text{s.t.} & x_1 \in X_1
\end{array}
$$

only x_1

$$
\begin{array}{ll}
\min & f_2(x_2) - \langle A_2^T \lambda + D_2^T \mu, x_2 \rangle \\
\text{s.t.} & x_2 \in X_2
\end{array}
$$

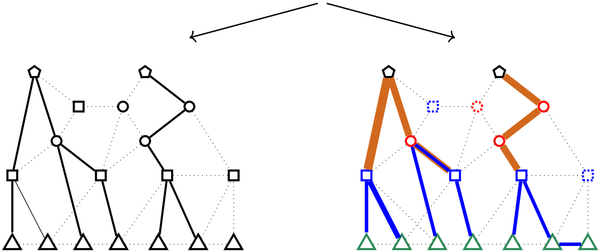
only x_2

- each subproblem covers only a part of the structure

Application to Wind Park Extension Planning

Idea: Decompose problem according to cost structure

- Relax constraints coupling decisions
  - which hubs/connections to be built
  - how many transformers/cables to be installed



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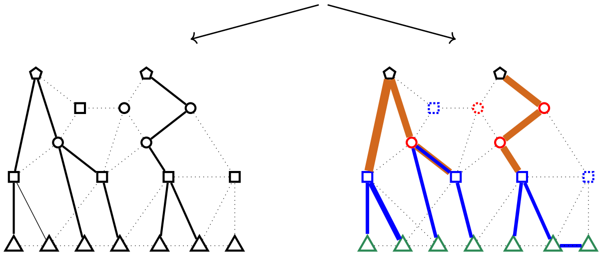

Fixed-charge network design

- only setup costs
- no flows

Topology Design

Network dimensioning

- only flow vars
- no setup costs

Flow + Capacity Dimensioning

Application to Wind Park Extension Planning

Idea: Decompose problem according to cost structure

Fixed-charge network design	Network dimensioning
▸ only setup costs ▸ no flows	▸ only flow vars ▸ no setup costs
Topology Design	**Flow + Capacity Dimensioning**

- Both subproblems are MIP
- In practise both MIP are enhanced using additional constraints
- Hard problems in theory
- ***Efficiently solvable in practise by state-of-the-art MIP solvers***

Advantages of this approach

- Model can be solved efficiently (if subproblems are easy enough)

$\hookrightarrow$ Bundle Methods, ConicBundle

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- Provide ***global bounds***

  Certify quality of heuristic solutions!

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In particular useful if models are too large for standard approaches.

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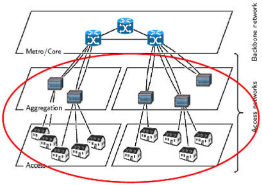
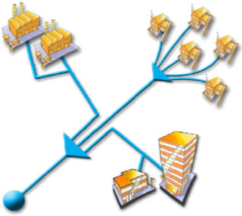
FFTx Network Design Problem



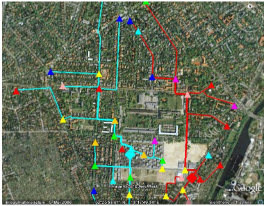


Passive optical access networks

- Replace copper lines by (shared) optical fibers
  - Higher bandwidth
  - Longer reach
  - Less energy consumption
- Practical difficulties
  - Requires new links $\Rightarrow$ Huge investments
  - Complex technologies
  - Long deployment process
  - Mix with old technologies
  - ...

Planning Problems

Potential fiber links



Partial FTTH network solution

Static network design

- Where to place nodes and links?
  - Central offices
  - Distribution points
- What components to install? (splitters, cases, cables, ducts pipes, . . .)

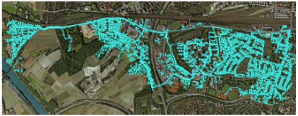
Network evolution planning

- When to build which region?
- Meet coverage & budget constraints

Planning Problems


Year 1


Year 2


Year 3

Static network design

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Network evolution planning

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FFTx and Wind Park Extension Planning

FFTx

- customers
- distribution points
- central offices
- splitters, . . .
- fibres
- capacities
- data transmission

Wind Park Ext. P.

- wind parks
- hubs
- on-shore grids
- transformers, . . .
- power lines
- capacities
- energy flow

Both problems lead to the same model
$\rightsquigarrow$ can be solved with the same methods

Computing primal solutions

Lagrangian decomposition: Relaxation technique

- $\hookrightarrow$ (Typically) incompatible subproblem solutions
- $\hookrightarrow$ Need heuristics to fix incompatibilities

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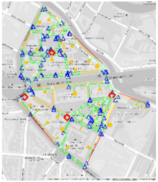
General primal heuristic

(1) Solve topology subproblem for current μ:
$\rightarrow$ ***Min network topology***.

(2) Fix (only) chosen edges and nodes
(or bias setup cost accordingly)

(3) Solve restricted full ILP $\rightarrow$ ***Full solution***
(permitting selection of additional nodes and edges)

Instances



instance a: TU Berlin campus

instance c: Ahorn

inst.	$\|V\|$	$\|E\|$	$\|V_H\|$	$\|V_O\|$	$\|V_W\|$	$\sum d_v$	$\overline{c_e}$	$\overline{c_{dp}}$	$\overline{c_{co}}$
a	637	826	97	4	36	488	5878	4600	418670
b	1315	1434	143	5	88	278	5341	4576	509750
c	1675	1730	99	5	552	2290	637	3433	413156
d	2271	1419	494	4	349	717	1039	1500	418670
e	6750	7352	520	11	571	5006	672	3186	305454
f	6750	7352	520	11	571	5006	672	3186	300000
g	4110	4350	224	6	1072	4164	635	3512	466927
h	4227	4484	314	5	1379	5542	3483	3417	477505
i	11544	12478	875	15	3862	14088	3326	3274	522729

- setup costs $\sim$ 5.000 $\times$ fiber cost per link
- $\frac{\text{fixed charge cost}}{\text{variable cost}} \sim \frac{18}{1} \dots \frac{150}{1}$ in solutions

Computational Results

Lagrangian Decomposition

- Stable, fast, few iters / evals
- Very good initial bound / sol

MIP+Cuts

- Bound improves too slow
- Poor initial solutions

	(FC)+(FP) Decomp.				Aggr. MIP + Cuts					Aggr. MIP		
P	*gap*	#It	#Ev	t [s]	*gap*	gap_{UB}	#Nodes	gap_L	$t_L[s]$	*gap*	gap_{UB}	#Nodes
a	0.97	8	10	576	0.24	0.23	51283	0.72	206	33.73	33.73	1294596
b	0.51	4	12	430	0.05	0.05	75182	0.34	107	35.46	35.39	1225815
c	1.65	6	10	512	1.15	1.05	124355	1.46	238	3.07	2.74	989764
d	1.23	7	10	273	0.93	0.93	103538	1.10	166	5.35	4.54	712821
e	3.34	9	19	4776	—	2.88	2952	—	>30*h*	—	26.52	214674
f	3.25	5	13	3362	—	2.76	2199	—	>30*h*	33.01	26.38	190280
g	2.36	8	10	2435	2.70	1.75	35673	2.92	13393	8.20	6.66	367052
h	0.56	6	10	1779	0.54	0.44	35938	—	3478	—	12.52	33733
i	0.73	3	4	2*h*	—	3.13	0	—	>30*h*	—	10.45	120503

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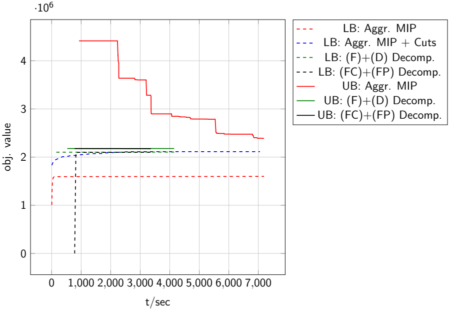
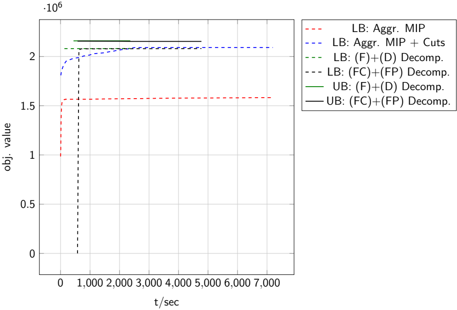


Bounds for Instance f



Bounds for Instance e

Conclusions

Summary

- Many planning problems lead to ***large scale*** combinatorial optimisation problems
  - Models often ***too large*** for standard solvers
- Lagrangian decomposition is a powerful tool to
  - ***Decompose*** the problem into independent subproblems
  - Compute valid ***global bounds***
  - Guide heuristics with ***provable quality guarantees***
  - Relatively short computation times
- Has been successfully applied in many areas:
  - telecommunication
  - railway and traffic planning
  - stochastic optimisation
  - ...

- Energy network design poses challenging problems to ***Mathematical Optimisation*** that could be tackled by ***Lagrangian relaxation***!

Thank you for your attention.
Questions?